SRR & CVR GOVT. DEGREE COLLEGE

(Autonomous) NAAC 'B+' Grade

DEPARTMENT OF MATHEMATICS

II B.Sc. MATHEMATICS

SEMESTER–IV, PAPER – IV

REAL ANALYSIS

MODEL QUESTION PAPER

Time: 3Hrs **Max.Marks:60**

SECTION-A

Answer any FIVE questions **5x4=20 M**

1. Show that every Cauchy sequence is convergent.

2. Prove that the sequence $\{S_n\}$, where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n}$ is convergent.

3. Test for the convergence of $\sum_{n=1}^{\infty} \frac{1}{2^n+3^n}$.

4. Show that the series $\sum(-1)^n\left(\sqrt{n^2+1}-n\right)$ is conditionally convergent.

5. If f:[a, b]→R is continuous on [a, b] then prove that f is bounded on [a, b].

6. Discuss the continuity of f defined by f(x) = $\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$, $x \neq 0$ and f(0)=1 at x=0.

7. Verify Cauchy's Mean Value Theorem for f(x)=x^2, g(x)=x^3 in [1, 2].

8. Discuss the applicability of Lagrange's mean value theorem for f(x)=x(x-1) (x-2) on $\left[0, \frac{1}{2}\right]$.

9. Prove that the function defined on [0, 1] by f(x) = 1 when x is rational, f(x) = - 1, when x is irrational, is not integrable.

10. Evaluate $\int_0^{\frac{\Pi}{4}}(sec^4 x - Tan^4 x)dx$.

SECTION – B

Answer ALL questions **5x8=40 M**

11. (a). State and Prove Cauchy's first theorem on Limits.

(OR)

(b). Show that the sequence $\{S_n\}$ defined by $S_n=1+\frac{1}{1!}+\frac{1}{2!}+\cdots\ldots+\frac{1}{n!}$ is convergent.

12. (a). State and prove D'Alembert's Ratio Test.

(OR)

(b). State and prove Leibnitz Test.

13. (a). Examine the continuity of the function 'f' defined by $f(x) = |x - 1| + |x - 2|$ at $x=1$ and 2.

(OR)

(b). If a function f is continuous on [a, b], then f is uniformly continuous on [a, b].

14. (a). State and prove Rolle's mean value theorem.

(OR)

(b). Show that $f(x) = |x| + |x - 1|$ is not derivable at x=0 and x=1.

15. (a). A bounded function $f:[a, b]\to R$ is Reimann integrable on [a, b] if and only if for each $\in > o$ there exists a partition P of [a,b] such that $U(p, f) - L(p, f) < \in$.

(OR)

(b). State and prove fundamental theorem of Integral Calculus.